Lid-driven cavity highly turbulent flow subjected to high magnetic field

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Determination of critical time-step for explicit MHD schemes

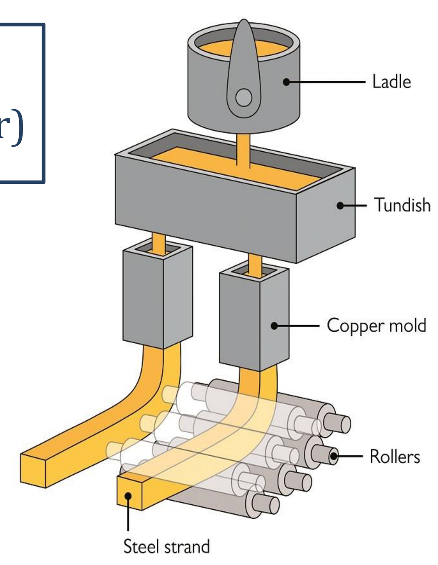
Control of fluid flow in CC:

- Electromagnetic **braking** and **stirring** in **continuous casting** process of steel
- Coupled simulation in commercial software:

THERCAST® (Fluid mechanical solver)

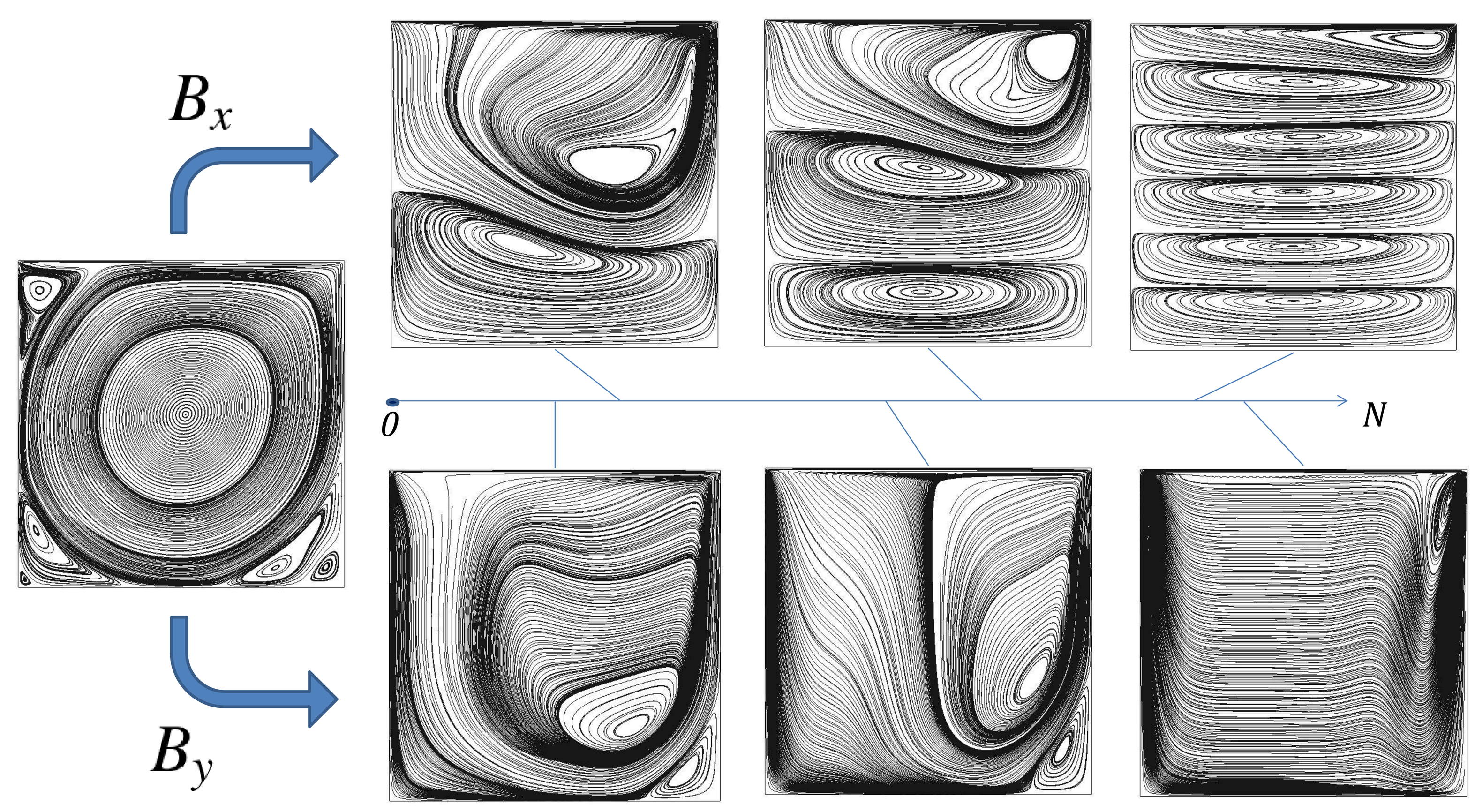
Lorentz forces

MATELEC (EM solver)



A representative study case:

- Problem:
  - Highly **turbulent flow**
- Solution:
  - Impose a constant magnetic field to control and **brake the flow**
- Numerical procedure:
  - Lid-driven cavity benchmark[1]
  - Relate the **Stuart number** (N) to the target braking level



Computational model:

Electromagnetic Field:

$\Delta\phi = \partial_z(u_x B_y - u_y B_x) \equiv 0$

Lorentz Forces:

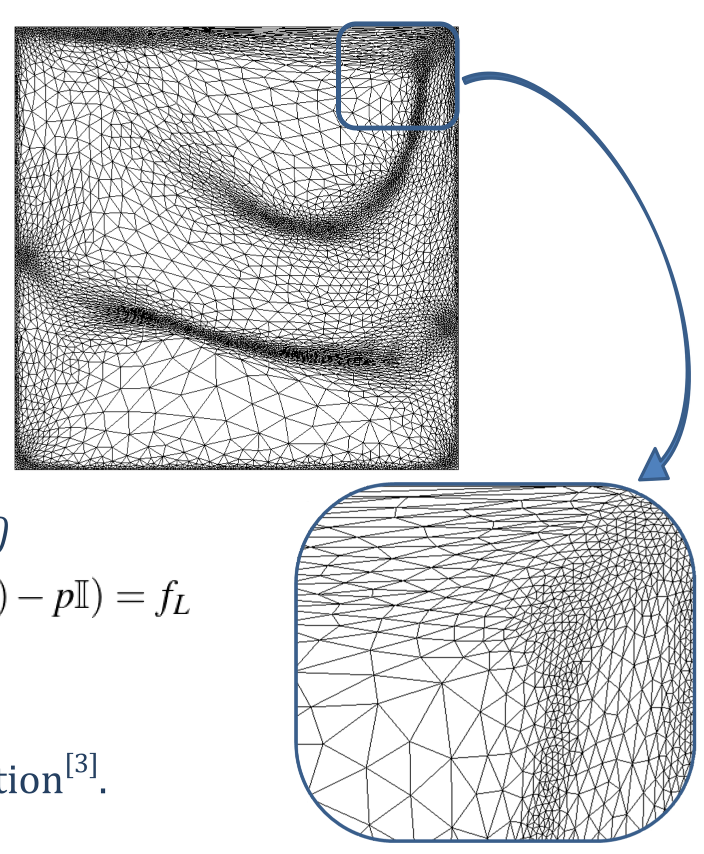
$f_L = \sigma(u \times B) \times B$

Navier-Stokes:

(multiscale stabilized FE[2])

$$\begin{cases} \rho(\partial u + u \cdot \nabla u) - \nabla \cdot (2\mu\varepsilon(u) - p\mathbb{I}) = f_L \\ \nabla \cdot u = 0 \end{cases}$$

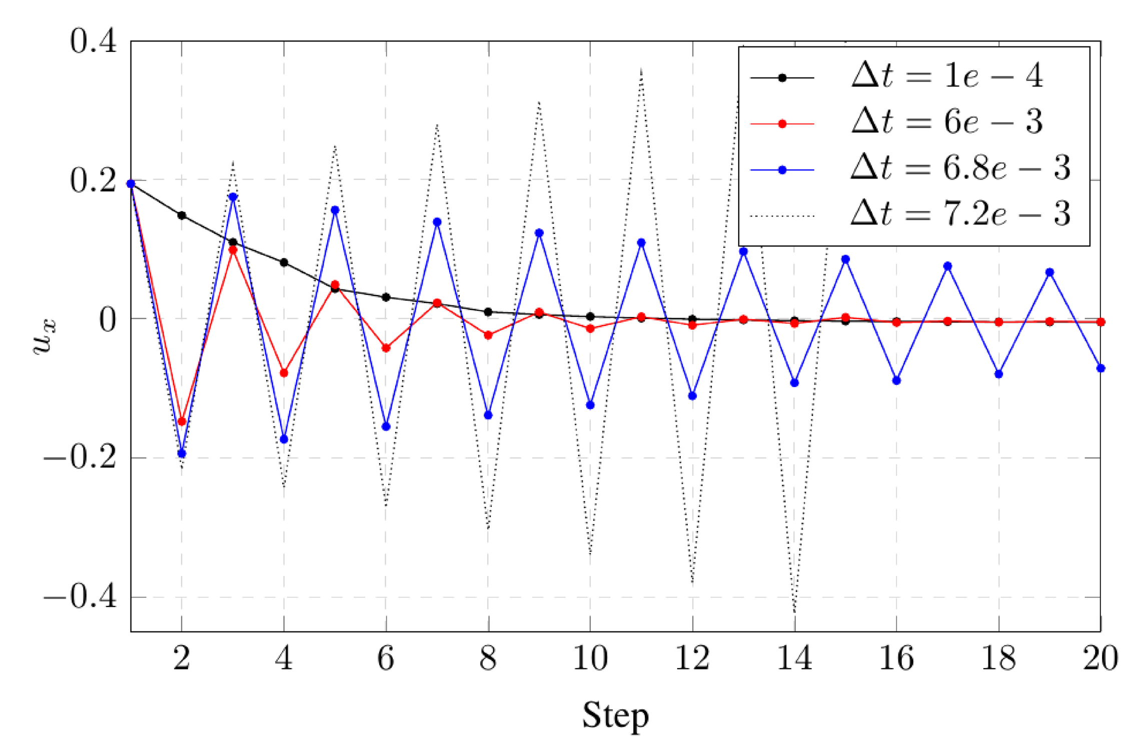
Anisotropic mesh adaptation[3].



Numerical instabilities:

- The Lorentz force is tracked **explicitly**.
- The excessive size of the time-step leads to completely brake the flow normally to B and to accelerate it in the opposite direction, i.e. $u^n \cdot f_L^n < 0$.
- The CFL condition is not capable anymore to guarantee **numerical convergence**.

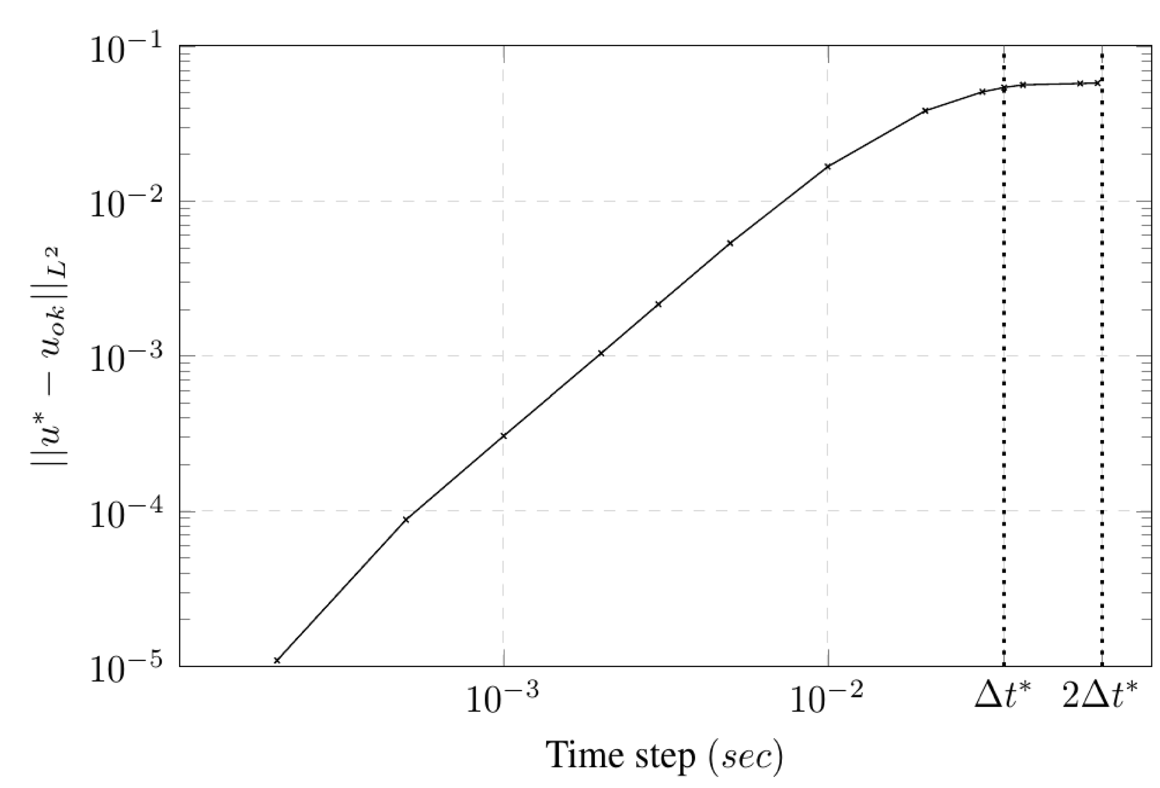
$$\Delta t_{CFL} < C\frac{h}{u}$$

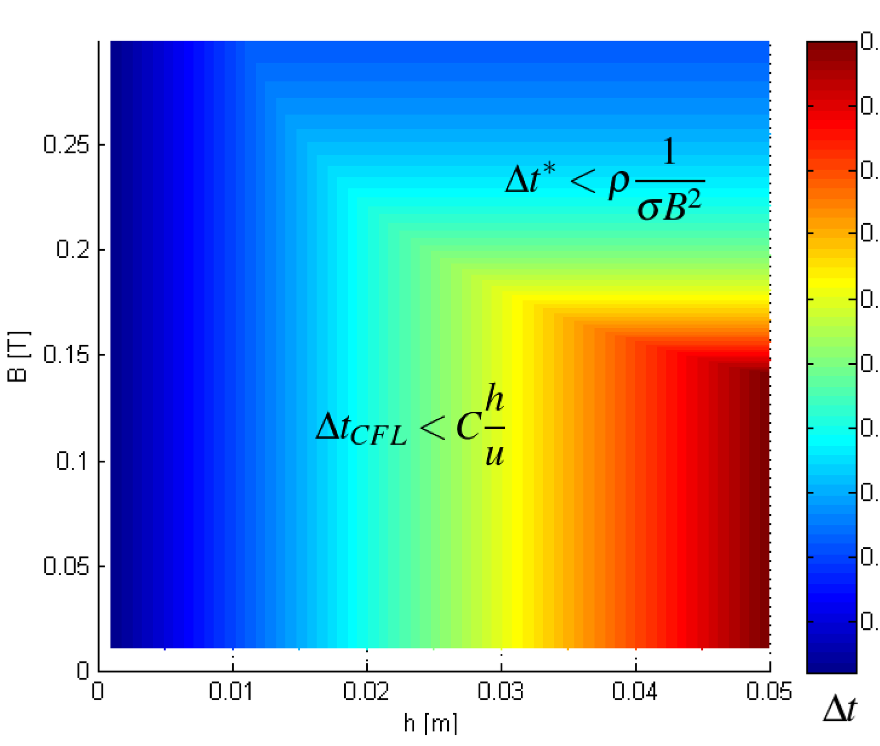


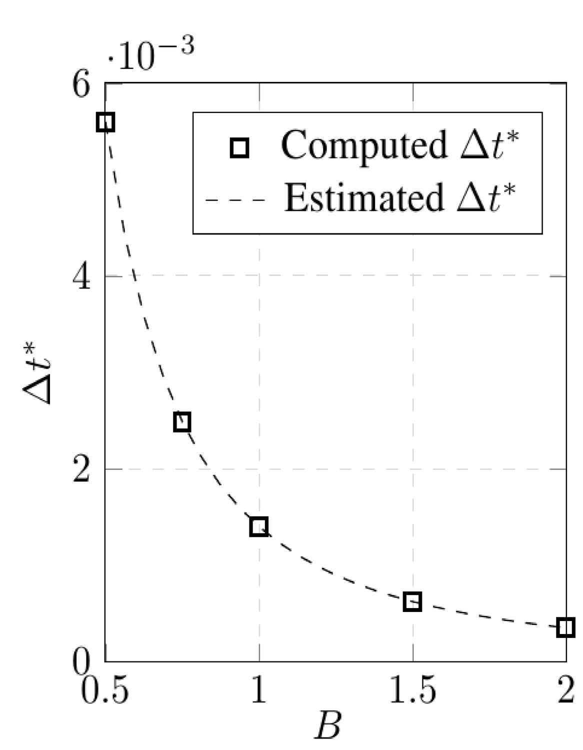
Determination of the critical time-step:

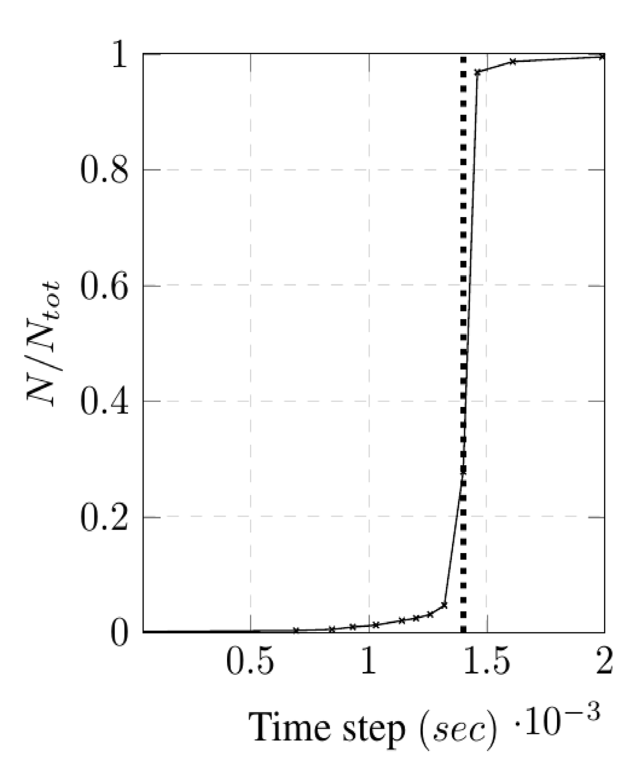
- Solution:
  - Computation of the **limit time-step** as the time-step which leads to a complete dissipation of the kinetic energy within one time increment when the magnetic field is normal to the velocity.

$$\Delta t^* < \rho\frac{1}{\sigma B^2}$$



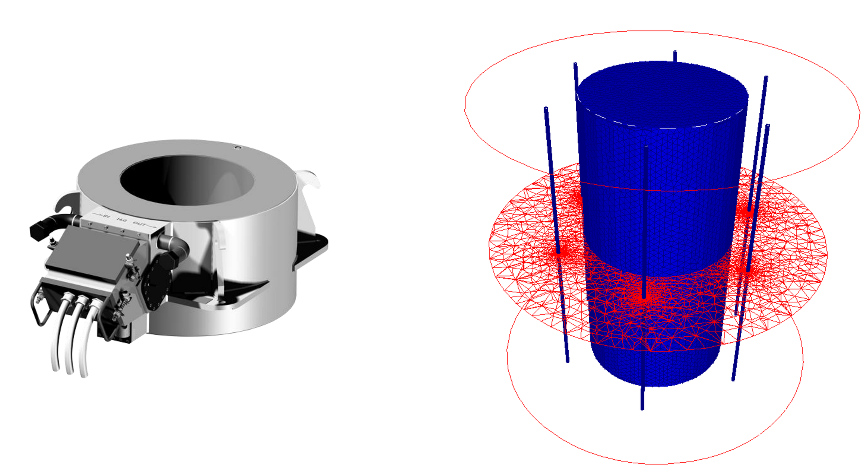






Conclusions & Perspectives:

- The flow is highly affected by the magnetic field and its direction.
- In some configurations, the CFL condition is not enough to guarantee the **numerical convergence**.
- A new **time-step threshold** which guarantees convergence in **explicit schemes** is proposed.
- An **implicit** modelling of the Lorentz force has to be included in the multiscale stabilized finite-elements solver.
- The braking effect will be taken into account in the complete simulation of **electromagnetic stirring.**



Acknowledgements:

The authors would like to gratefully thank Transvalor for funding this research and for the continuous support which allowed the full exploitation of its software's capabilities.

References:

[1] U. Ghia, K. N. Ghia, and C. Shin. High-resolution for incompressible flow using the Navier-Stokes equations and a multigrid method. Journal of computational physics, 48(3):387–411, 1982.

[2] E. Hachem, B. Rivaux, T. Kloczko, H. Digonnet, and T. Coupez. Stabilized finite element method for incompressible flows with high Reynolds number. Journal of computational physics, 229:8643–8665, 2010.

[3] T. Coupez and E. Hachem. Solution of high-Reynolds incompressible flow with stabilized finite element and adaptive anisotropic meshing. Computer Methods in Applied Mechanics and Engineering, 267:65–85, 2013.

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